Quiz 4 (20 points)

You may use computers to check, but you most show the calculus you did, including all steps to find the following.

(1) Given $f(x)=\sqrt{\dfrac{x^2\cos x}{2x+3}}$, find $f'(x)$ (7 points)

(Note: there is an easier way to do this than just directly... the computer does not always show you the easiest way)

Use logarithmic differentiation: Find $\frac{dy}{dx}$

$$\ln y = \ln\sqrt{\frac{x^2\cos x}{2x+3}}$$

$$\ln y = \frac{1}{2}\left[\ln(x^2\cos x) - \ln(2x+3)\right]$$

$$\frac{d}{dx}\ln y = \frac{d}{dx}\left[\frac{1}{2}\left(2\ln x + \ln\cos x - \ln(2x+3)\right)\right]$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{1}{x} + \frac{1\cdot -\sin x}{2\cos x} - \frac{1}{2}\frac{2}{2x+3}$$

$$\frac{dy}{dx} = y\left(\frac{1}{x} - \frac{1}{2}\tan x - \frac{1}{2x+3}\right)$$

$$\boxed{\frac{dy}{dx} = \sqrt{\frac{x^2\cos x}{2x+3}}\left(\frac{1}{x} - \frac{1}{2}\tan x - \frac{1}{2x+3}\right)}$$

(2) Differentiate: $y=\sin^{-1}(2x)+x\sqrt{1-x^2}$ (6 points)

product and chain

$\left(\frac{d}{dx}[\sin^{-1}x]=\frac{1}{\sqrt{1-x^2}}\right)$

$$y'=\frac{1}{\sqrt{1-(2x)^2}}\frac{d}{dx}[2x]+\sqrt{1-x^2}+x\cdot\frac{1}{2}(1-x^2)^{-1/2}(-2x)$$

$$\boxed{y'=\frac{2}{\sqrt{1-4x^2}}+\sqrt{1-x^2}-\frac{x^2}{\sqrt{1-x^2}}}$$

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(3) Evaluate: $\int\frac{1}{2+9x^2}\,dx$ (7 points)

Using

$$\int\frac{1}{a^2+x^2}\,dx=\frac{1}{a}\tan^{-1}\frac{x}{a}+C$$

$u=3x$

$$\frac{1}{3}\int\frac{1}{2+u^2}\,du$$

$$\frac{1}{3}\cdot\frac{1}{\sqrt{2}}\tan^{-1}\frac{u}{\sqrt{2}}+C$$

$$\boxed{\frac{1}{3\sqrt{2}}\tan^{-1}\left(\frac{3x}{\sqrt{2}}\right)+C}$$